$$\begin{aligned} |G| &= p^k r \Rightarrow \exists H \subseteq G, |H| = p^k \\ |G| &= p^n m, p \not| m, P_i \subseteq G, |P_i| = p^n \Rightarrow \\ P_1 &= g P_2 g^{-1}, H \subseteq P_i \\ \sum_i 1 &= s \Rightarrow s | m, s \equiv 1 \pmod{p} \end{aligned}$$